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Reduction formula for

$\int e^{ax} \cos^n x \, dx$.

Here, $\int e^{ax} \cos^n x \, dx$

$= \cos^n x \int e^{ax} dx - \int \left[\frac{d}{dx}(\cos^n x) \int e^{ax}\right] dx$

$= \frac{e^{ax}}{a} \cos^n x - \int n \cos^{n-1} x \left(-\sin x\right) \frac{e^{ax}}{a}$

$= \frac{1}{a} e^{ax} \cos^n x + \frac{n}{a} \int \left(\cos^{n-1} x \sin x\right) e^{ax} dx$

$= \frac{1}{a} e^{ax} \cos^n x + \frac{n}{a} \left(\cos^{n-1} x \cdot \sin x\right) \int e^{ax}$

$- \frac{n}{a} \int \frac{d}{dx} \left(\cos^{n-1} x \cdot \sin x\right) \int e^{ax} dx$.

$= \frac{1}{a} e^{ax} \cos^n x + \frac{n}{a} \cos^{n-1} x \cdot \sin x \frac{e^{ax}}{a}$

$- \frac{n}{a} \int \Big[ -(n-1) \cos^{n-2} x \cdot \sin x \cdot \sin x +$

$\cos^{n-1} x \cdot \cos x \Big] \frac{e^{ax}}{a} dx$.

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$= \frac{1}{a} e^{ax}\cos^n x + \frac{n}{a^2} e^{ax}\cos^{n-1}x \sin x$

$- \frac{n}{a}\int \left[-(n-1)\cos^{n-2}x\,(1-\cos^2 x) + \cos^n x\right] \frac{e^{ax}}{a}\,dx$

$= \frac{1}{a} e^{ax}\cos^n x + \frac{n}{a^2} e^{ax}\cos^{n-1}x \sin x$

$+ \frac{n(n-1)}{a^2}\int e^{ax}\cos^{n-2}x\,dx - \frac{n^2}{a^2}\int e^{x}\cos^n x\,dx$

Or. $\left(1+\frac{n^2}{a^2}\right)\int e^{ax}\cos^n x\,dx$

$= \frac{1}{a^2} e^{ax}(a\cos x + n\sin x)\cos^{n-1}x$

$+ \frac{n(n-1)}{a^2}\int e^{ax}\cos^{n-2}x\,dx$.

This is the required reduction formula.

<u>Note</u> If $I_n = \int e^{ax}\cos^n x\,dx$ then

$(a^2+n^2) I_n = e^{ax}(a\cos x + n\sin x)\cdot\cos^{n-1}x + n(n-1) I_{n-2}$